Measuring relative phase between two waveforms using an oscilloscope

Overview

There are a number of ways to measure the phase difference between two voltage waveforms using an oscilloscope. This document covers four methods and summarizes the advantages and limitations of each.

Method	Oscilloscope Requirements	Waveform Requirements	Advantages	Limitations
Time-difference	2 channels		B	a
Lissajous	1 v. 2 mode	Sinusoidal only	D, E	a, c
Product	1 × 2 mode	Sinusoidal only	A	a, b
Curvefitting	2 channels Data connectivity		A, B, C, D	d

a: Errors introduced when dc offsets are present
b: Reduced precision near $\theta = 0°, 180°$
c: Reduced precision near $\theta = 90°, 270°$
d: Possible incorrect solution

A: No manual reading of values from display (can be automated)
B: Works with nonsinusoidal waveforms
C: Uses entire waveform information to increase accuracy
D: Cool
E: No need to measure time scales

Explanations are given to show how each method works. These are given for reference, but understanding them is not necessary to apply the methods.

Measuring relative phase between oscilloscope traces using the time-difference method

Requirements:

Oscilloscope:	Waveforms:
• Two channels (Includes virtually all oscilloscopes.)	• *Common frequency and shape*

Method:

- **Display** both channels as a function of time.
- **Scale each voltage channel** so that each waveform fits in the display.
- **Ground** or zero each channel separately and adjust the line to the center axis of the display.
- Return to **ac coupling**..
- (Optional) If you can continuously adjust the voltage per division, **scale** your waveforms to an even number of divisions. You can then set the zero crossing more accurately by positioning the waveform between gridlines on the display.
- **Pick a feature** (e.g., peak or zero crossing for sinusoids, rising or falling transition for square waves) to base your time measurements on. The peak of a sinusoid is not affected by dc offsets, but is harder to pinpoint than the zero crossing.

Then follow one of the methods below:	
Method 1	**Method 2** (requires continuous time base scaling)
• **Measure the period** T between repeats. Digital scopes often measure $f = 1 / T$ automatically. • **Measure** t_d, the smallest time difference between occurences of the feature on the two waveforms. • The phase difference is then $\|\theta_2 - \theta_1\| = 360° \frac{t_d}{T}$	• **Fit** one period of your waveform to 4, 6, or 9 divisions. • **Scale** the time base by a factor of ten (expand the plot horizontally), so that each division will be 9, 6, or 4°, respectively. • **Count** the number of divisions between similar points on the two waveforms.

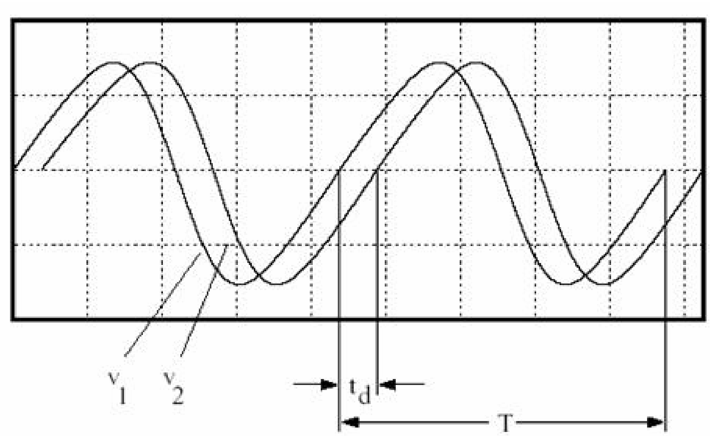

Figure 1. Dual-channel display. With either method, the sign of θ is determined by which channel is leading (to the left of) the other. In the figure, v_1 leads v_2.

Measuring relative phase between oscilloscope traces using the Lissajous (ellipse) method

Requirements:

Oscilloscope:	Waveforms:
• Able to display the voltage of one channel vertically and the other channel horizontally. (Includes virtually all oscilloscopes.)	• *Sinusoidal* • *Common frequency*

Method:

- Set the oscilloscope to ***xy* mode**.
- **Scale** each voltage channel so that the ellipse fits in the display. (This may be a line if the phase difference is near 0° or 180°.)
- **Ground** or zero each channel separately and adjust the line to the center (vertical or horizontal) axis of the display. (On analog scopes, you can ground both simultaneously and center the resulting dot.)
- Return to **ac coupling** to display the ellipse.
- (Optional) If you can continuously adjust the voltage per division, **scale** your waveforms to an even number of divisions. You can then center the ellipse more accurately by positioning the waveform between gridlines on the display.
- **Measure** the horizontal width *A* and zero crossing width *C* as shown in the figure to the right.
- The magnitude of the phase difference is then given by

$$|\theta_2 - \theta_1| = \begin{cases} \pm \sin^{-1}(C/A) & \text{top of ellipse in QI} \\ \pm\left[180° - \sin^{-1}(C/A)\right] & \text{top of ellipse in QII} \end{cases}$$

- The sign of $(\theta_2 - \theta_1)$ must be determined by inspection of the dual-channel trace.

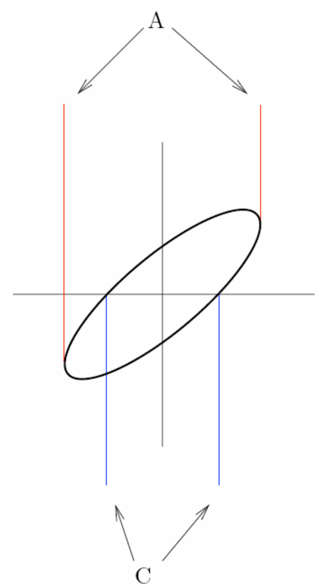

Figure 2. Lissajous figure. (by Paul Kavan)

Explanation:

Given two waveforms: v_1 vertical, v_2 horizontal	$v_1(t) = V_{1p}\cos(\omega t + \theta_1)$ $v_2(t) = V_{2p}\cos(\omega t + \theta_2)$
The ellipse will cross the horizontal axis at time t_0 when $v_1(t_0) = 0$ or	$\omega t_0 + \theta_1 = \left(n + \frac{1}{2}\right)\pi \Rightarrow t_0 = \frac{1}{\omega}\left[\left(n + \frac{1}{2}\right)\pi - \theta_1\right]$
At this time the value of $v_2(t_0)$ will be	$v_2(t_0) = V_{2p}\cos\left[\left(n + \frac{1}{2}\right)\pi - \theta_1 + \theta_2\right]$
A trig identity yields* $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	$\underbrace{\frac{v_2(t_0)}{V_{2p}}}_{\substack{C/A \\ \text{above}}} = \pm\sin(\theta_2 - \theta_1)\begin{cases} + \text{ if } n \text{ is odd} \\ - \text{ if } n \text{ is even}\end{cases}$

Now we have two nasty details to take care of.

- *First, we have to be careful taking the inverse sine, since a phase change between 90° and 180° gives rise to the same (C / A) ratio as its coangle.*

To decide which we have, find the times t_{p1} and t_{p2} when $v_1(t)$ and $v_2(t)$ peak:	$\omega t_{p1} + \theta_1 = 0 \Rightarrow t_{p1} = -\frac{\theta_1}{\omega}$
And look at $v_2(t)$ at that time:	$v_2(t_{p1}) = V_{2p}\cos(\theta_2 - \theta_1)$
Conversely	$v_1(t_{p2}) = V_{1p}\cos(\theta_1 - \theta_2) = V_{1p}\cos(\theta_2 - \theta_1)$

So if v_2 is positive when v_1 is maximum positive if $|\theta_2 - \theta_1|$ is between 0° and 90°. For this case, the top and right side of the ellipse will be in Quadrant I.

But if v_2 is negative when v_1 maximum positive, then we need the other angle with the same sine. This means the top of the ellipse will be in Quadrant II and the right side in Quadrant IV. So if $|\theta_2 - \theta_1| > 90°$, then the actual inverse sine is $\left[180° - \sin^{-1}(C/A)\right]$, where $\sin^{-1}$ represents the principle inverse sine between 0° and 90°.

- *Second, what is the sign of* $(\theta_2 - \theta_1)$?

Suppose we have a case where two voltages $v'_1(t)$ and $v'_2(t)$ have the same amplitudes as before but with opposite phase angles:	$v'_1(t) = V_{1p}\cos(\omega t - \theta_1)$ $v'_2(t) = V_{2p}\cos(\omega t - \theta_2)$
Since $\cos(\alpha) = \cos(-\alpha)$,	$v'_1(t) = V_{1p}\cos[\omega(-t) + \theta_1]$ $v'_2(t) = V_{2p}\cos[\omega(-t) + \theta_2]$

This is the exact same as $v_1(t)$ and $v_2(t)$, but reversed in time. The Lissajous figure will look exactly the same, but the trace will precess in the opposite direction. That means *the sign of the phase difference cannot be determined in xy mode*.

So **we need the dual-channel trace** to determine the sign of $(\theta_2 - \theta_1)$.

Measuring relative phase between oscilloscope traces using the product method

Requirements:

Oscilloscope:	Waveforms:
• Automatic amplitude measurement (preferably rms value) for each channel. • Display the product of the two channels and calculate its dc offset automatically. This method was developed using the Tektronix 2012B oscilloscope.	• *Sinusoidal* • *Common frequency*

Method:

- **Display the two traces** in voltage v. time mode, along with the **product trace** in voltage^2 v. time mode. Use **ac coupling**.
- **Fit** the traces in the screen vertically.
- **Display the MEAN** value (dc offset $v_{math,dc}$) of the product trace. Show about 10 of its periods to ensure that the calculation is not affected by partial waveforms at the beginning and end of the trace. You may also want to average the sampling to reduce error.
- **Display the rms amplitude** of each of the channels (V_{1rms} and V_{2rms}). Using rms (rather than peak-to-peak) means that the scope has averaged over the waveform.
- The phase difference is then*

$$\cos(\theta_2 - \theta_1) = \frac{v_{math,dc}}{V_{1rms}V_{2rms}} = \frac{2v_{math,dc}}{V_{1p}V_{2p}} = \frac{8v_{math,dc}}{V_{1p-p}V_{2p-p}}$$

- The sign of $(\theta_2 - \theta_1)$ still has to be determined by inspection of the dual-channel trace.

Explanation:

Given two waveforms:	$v_1(t) = V_{1p}\cos(\omega t + \theta_1)$ $v_2(t) = V_{2p}\cos(\omega t + \theta_2)$
Their product is	$v_{math}(t) = V_{1p}V_{2p}\cos(\omega t + \theta_1)\cos(\omega t + \theta_2)$
Applying the trig identity	$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$
yields	$v_{math}(t) = \frac{V_{1p}V_{2p}}{2}\left[\cos(\theta_1-\theta_2)+\cos(2\omega t+\theta_1+\theta_2)\right]$
The phase difference determines the dc offset of the math trace:	$v_{math,dc} = \frac{V_{1p}V_{2p}}{2}\cos(\theta_1-\theta_2)$

*This method is least precise when the phase difference is nearly 0° or 180°, cases where $\cos\theta$ is not sensitive to θ.

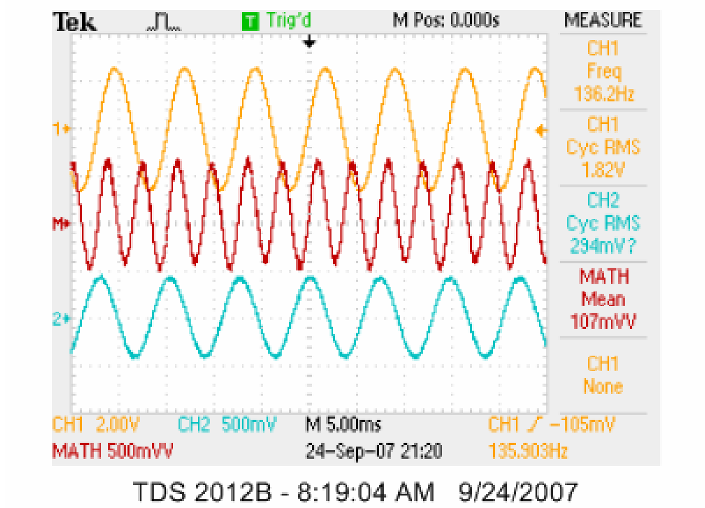

Figure 3. Oscilloscope display for the product method showing the dual-channel display and the product trace, along with the values for V_{1rms}, V_{2rms}, and $v_{math,dc}$ as calculated by the oscilloscope. Here $|\theta_2 - \theta_1| = \cos^{-1}[0.107/(1.82 \cdot 0.294)] = 78.5°$. Since the peak in channel 2 is to the left of the peak in channel 1, v_2 leads v_1.

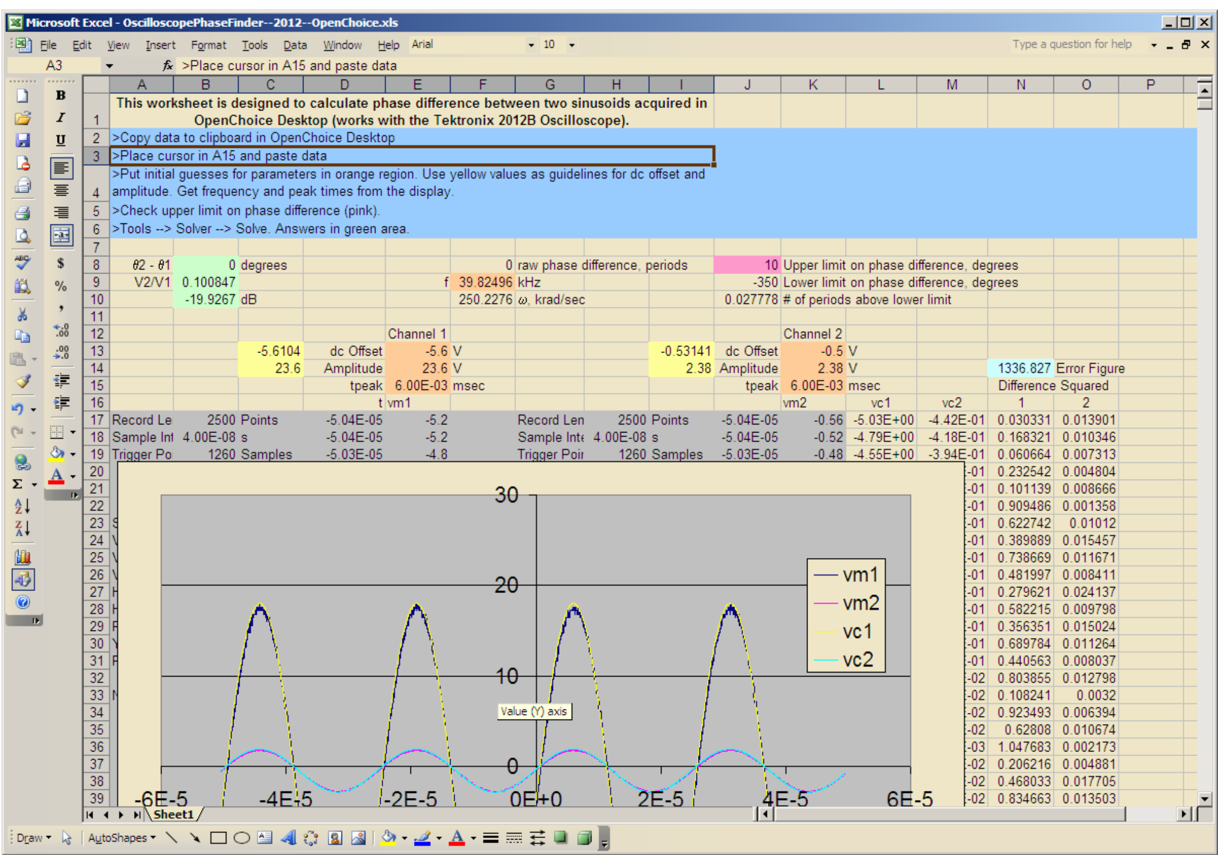

Figure 4. Spreadsheet used for curvefitting method. Regions are color coded according to function: instructions, preference, data, results, error figure (between guess and experiment), parameter guesses (calculated from data), fit parameters. Approximate values of the fit parameters must supplied before using the Solve function.

Measuring relative phase between oscilloscope traces using the curvefitting method

Requirements:

Oscilloscope:	Waveforms:
• Dual channel capability • Capability to transfer data points to a computer This method was developed using the Tektronix 2012B oscilloscope. It has been implemented using OpenChoice Desktop software and an Excel spreadsheet called "OscilloscopePhaseFinder--2012--OpenChoice.xls"	• *Common frequency and shape*

Method:

- **Display** the two traces in voltage v. time mode.
- **Scale the display** to show at least one full period and maximize the waveform sizes without clipping.
- **Open** the OpenChoice Desktop software.
- **Acquire** the display to the computer and **copy** it to the clipboard.
- **Open the spreadsheet** mentioned above (or create your own) and **paste** the data into the appropriate place.
- In the curvefitting spreadsheet or other software, **set initial guesses** for the amplitudes V_{1p} and V_{2p}, dc voltage offsets v_{dc1} and v_{dc2}, and peak times t_{p1} and t_{p2} for each channel, as well as the frequency.
- **Initiate the fit routine.** The spreadsheet or other software must then **calculate** voltage v. time for each channel using the above parameters and compare those values with the measured voltages. It then needs to **seek values** for the above parameters that produce the closest match between the calculated and measured values.
- Once the peak times are known for each channel, the software can compute the phase difference according to*

  $$\theta_2 - \theta_1 = \omega\left(t_{p1} - t_{p2}\right),$$

  where t_{p1}, t_{p2}, and ω are results taken from the spreadsheet.
- This can be remapped into a desired range by adding or subtracting multiples of 360°, depending on the application. For instance, a second-order lowpass filter would usually have phase ranging from 0° (at low frequency) to –360° (the asymptotic value at high frequency).

Explanation:

Given two waveforms:	$v_1(t) = v_{dc1} + V_{1p}\cos(\omega t + \theta_1)$ $v_2(t) = v_{dc2} + V_{2p}\cos(\omega t + \theta_2)$
The waveforms peak at times t_{p1} and t_{p2}:	$\omega t_{p1} + \theta_1 = 0$ $\omega t_{p2} + \theta_2 = 0$
Substracting the lower equation from the upper,	$\theta_2 - \theta_1 = \omega\left(t_{p1} - t_{p2}\right)$

*Note that dc offsets in the signal don't cause inaccurate phase measurements—they are accounted for in the fit procedure.